Unit 2: Traffic Behaviour and Traffic Theory Fundamentals

Module 2-1

The Stochastic Nature of Traffic Behaviour

Traffic Management Training Module

Austroads

Today's presenter

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Outline of this Module

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- Probabilistic aspect of traffic flow
- Stochastic queuing

Sections 3 and 4 of Guide to Traffic Management Part 2: Traffic Theory Concepts Austroads (2020)

Probabilistic Aspects of Traffic Flow

Traffic behaviour is influenced by a wide range of factors:

- Each vehicle is unique
- Each driver is unique
- Unique origin, destination, route, departure time, mode
- Effect of traffic control measures
- Weather conditions
- While many aspects of traffic behaviour may be stochastic in nature, they are not necessarily random.

Queuing

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Queues are everywhere:

- exiting a door at the end of an event
- a hamburger at McDonald’s, a haircut at the barber, shopping checkout
- boarding a bus, train, or plane
- traffic lights and ramp meters; motorway bottlenecks
- industrial plants; retail stores; service oriented industries; assembly lines

The same logic (with different parameters) can be applied to all these phenomena.

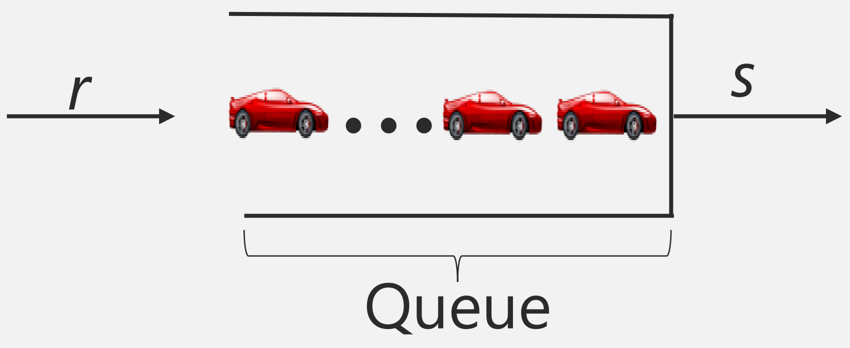
Queuing Theory

Queuing systems specification

- Arrival pattern (Deterministic or Stochastic)
- Queueing method (FIFO, LIFO, random, priority)
- Service configuration (single queue, parallel queues (i.e. multi-channel))
- Departure pattern (Deterministic or Stochastic)

Key Variables of Queuing Theory

- Arrival rate: r
- Departure rate: s
- Traffic intensity (Utilization factor) ρ = r/s
- Undersaturated: 0<ρ<1 or r > s
- Oversaturated: ρ>1 or r < s



Summary of Queuing Theory Formulae

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Description	Eqn no.	Equation[1]
Utilisation factor (ratio of arrival and service rates)	4.1	$\rho = r/s$
Probability of the system being empty	4.2	$P_0 = 1 - \rho$
Probability of exactly n units in the system	4.3	$P_0 = (1 - \rho)\, \rho^n$
Expected or average number of units in the system, including unit in service	4.4	$E(n) = \frac{\rho}{1-\rho} = \frac{r}{s-r}$
Probability of more than n units in the system	4.5	$Pr(n > N) = \rho^{N+1}$
Expected or average number of units in the system, excluding unit in service	4.6	$E(m) = \frac{\rho^2}{1-\rho} = \frac{\rho}{1-\rho} - \rho$
Relationship between expected numbers in the system including and excluding unit in service	4.7	$E(m) = E(n).\ \rho = E(n) - \rho$
Variance of expected number in the system, including unit in service	4.8	$\sigma^2(n) = \frac{\rho}{(1-\rho)^2}$
Probability of a zero waiting time before start of service	4.9	$Pr(w = 0) = P_0 = 1 - \rho$
Probability of wait greater than zero but not greater than w before start of service	4.10	$Pr(0 < wait \leq w) = \rho - \rho e^{-(s-r)w}$

Table 4.1 of Guide to Traffic Management Part 2: Traffic Theory Concepts Austroads (2020)

Summary of Queuing Theory Formulae

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Description	Eqn no.	Equation[1]
Probability of wait greater than w before start of service	4.11	$\Pr(\text{wait} > w) = \rho e^{-(s-r)w}$
Expected or average waiting time before start of service	4.12	$E(w) = \dfrac{\rho}{s-r} = \dfrac{r}{s(s-r)}$
Expected or average waiting time before start of service for those with a non-zero wait	4.13	$E(w \mid w > 0) = \dfrac{E(w)}{\rho} = \dfrac{1}{s-r}$
Expected or average total time in the system, including service time	4.14	$E(\tau) = \dfrac{1}{s-r}$
Relationship between expected total time in the system and expected waiting time before start of service	4.15	$E(\tau) = E(w) + \dfrac{1}{s}$

Table 4.1 of Guide to Traffic Management Part 2: Traffic Theory Concepts Austroads (2020)

Example Application of Queuing Theory

At a major sporting venue, patrons arrive in motor vehicles. At one of the entrances to the car parking area surrounding the venue, a single line of vehicles approaches the manually operated entry gate, where a cash payment is required to gain entry. The process of stopping at the payment point, offering payment, receiving change if necessary and departing the payment point averages 12 seconds per vehicle. Vehicles are arriving randomly at an average rate of 216 vehicles per hour. Appropriate design of the entry arrangements requires knowledge of the following values:

1. the queue storage length required prior to the payment point, assuming that the length provided must be adequate for at least 95% of the time

2. the average total delay, including time spent in the payment process, to vehicles entering the parking area at this gate.

Example Application of Queuing Theory

Value (1), the required queue storage length, is determined by the smallest value of N for which n, the number of vehicles in the queuing system, satisfies:

$$\Pr(n > N) = \rho^{N+1} \leq 0.05$$

the average arrival rate of vehicles, r, is 216 veh/h;

the average service time of 12 s/veh means that the average service rate, s, is 3600/12=300 veh/h;

therefore, ρ = r/s = 0.72 and

$$\Pr(n > 8) = 0.72^9 = 0.052$$
$$\Pr(n > 9) = 0.72^{10} \leq 0.037$$

Example Application of Queuing Theory

Given that n includes the vehicle occupying the payment point, space must be provided for 8 vehicles to queue before the payment point.

If a length of 6 m is allowed for each queued vehicle, a storage length of at least 48 m should be provided.

Value (2), the average total delay to entering vehicles, including time spent in the payment process, is given by Equation 4.14 as:

$$E(\tau) = \frac{1}{s-r} = \frac{3600}{300-216} = 42.9 \text{ s/veh}$$

Time to Reflect

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Q1. A right turn bay has an arrival rate of 8 cars per minute and departure rate of 10 cars per minute. The bay can accommodate 3 vehicles at maximum.

(a) **What is the probability of zero vehicle in the bay?**

(b) **What is the probability of 1 vehicle in the bay?**

(c) **What is the probability of 2 vehicles in the bay?**

(d) **What is the probability of 3 vehicles in the bay?**

(e) **What is the probability of spillover?**

Time to Reflect

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Q1. A right turn bay has an arrival rate of 8 cars per minute and departure rate of 10 cars per minute. The bay can accommodate 3 vehicles at maximum. What is the probability of n vehicle in the bay?

Explanation (a-d):

The probability of exactly n vehicles in the system is given by Equation 4.3 as:

$$P_n = (1 - \rho)\rho^n$$

In this case $\rho = \frac{8}{10} = 0.8$.

$$P_0 = (1 - \rho)\rho^0 = 0.2$$
$$P_1 = (1 - \rho)\rho^1 = 0.16$$
$$P_2 = (1 - \rho)\rho^2 = 0.128$$
$$P_3 = (1 - \rho)\rho^3 = 0.1024$$

Time to Reflect

Q1. A right turn bay has an arrival rate of 8 cars per minute and departure rate of 10 cars per minute. The bay can accommodate 3 vehicles at maximum. What is the probability of spillover?

Explanation (e):

The probability of spillover (aka spillback) is equivalent to have more than 3 vehicles in the bay which is $1-(P_0+P_1+P_2+P_3)$.

In this case, the probability of spillover is $1-(0.2+0.16+0.128+0.1024)$=$0.4096$~$41\%$.

References

Austroads (2020). Guide to Traffic Management Part 2: Traffic Theory Concepts. AGTM02-20, Austroads, Sydney, NSW. https://austroads.com.au/publications/traffic-management/agtm02/media/AGTM02-20-Part-2-Traffic-Theory-Concepts.pdf

Thank you for participating

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